#### Reminder

TA's announcement on course project report I

- PRA4 due 3/14
- HW4 due 3/21
- Course project progress report 2 due 3/26
- Come to OH for course project discussion!

Artificial Intelligence Methods for Social Good Lecture 16: Influence Maximization and Case Study on HIV Prevention Among Homeless Youth

> Instructor: Fei Fang <u>feifang@cmu.edu</u>



#### Influence Maximization Problem

Discussion

- Monte Carlo Tree Search
- Case Study: HIV Prevention Among Homeless Youth

# Learning Objectives

- Understand the concept of
  - Submodular function
- Describe
  - Independent Cascade Model
  - Linear Threshold Model
  - Influence Maximization Problem
  - Greedy Algorithm for Influence Maximization Problem
- For the case study, briefly describe
  - Significance/Motivation
  - Task being tackled, i.e., what is being predicted/estimated
  - Data usage, i.e., what data is used and how it is processed
  - Domain-specific considerations
  - AI method used
  - Evaluation process and criteria

### Social Networks



#### Social Networks

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# **Propagation Process**

- Viral propagation
  - Virus/Rumors
  - Get infected immediately and spread automatically
  - Individual agent does not make decisions

Is it really the case?

- Decision-based models
  - Individual agent makes decisions
  - Influence and adoption





https://sloanreview.mit.edu/article/the-power-of-product-recommendation-networks/

# **Propagation Process**

- General operational view:
  - A social network is represented as a (un)directed graph



- Nodes start either active or inactive
- Active node may trigger activation of neighboring nodes
- Monotonicity assumption: active nodes never deactivate

### Influence Response Function

# Influence Response Function

- Independent Draws
  - $\blacktriangleright$  *n* friends recommend it to me

► 
$$P(n) = 1 - (1 - p)^n$$

Diminishing return (concave function)



### **Influence Response Function**

# Influence Response Function

- Independent Draws
- Linear Threshold
  - Many of my friends bought the item, reaching a critical mass

$$\blacktriangleright P(b) = \delta(b > b_0)$$



# Influence Propagation Models

- Independent Cascade Model (Goldenberg, 2001)
  - Initial set of active nodes
  - Discrete time steps
  - When a node v just becomes active (activated in the last time step), it has a single chance of activating each currently inactive neighbor w (if failed, no second trial)
  - The activation attempt succeeds with probability  $p_{v,w}$
  - Process runs until no more activations possible

# Independent Cascade Model

#### Independent Cascade Model (Goldenberg, 2001)

Initial set of active nodes  $A_0$ For  $t = 1 \dots T$   $A_t \leftarrow \emptyset$ For  $v \in A_{t-1}$ For  $w \in neighbor(v)$  and  $w \notin \bigcup_{\tau=0}^t A_{\tau}$ If  $rand(\cdot) < p_{v,w}$ , then  $A_t \leftarrow A_t \cup \{w\}$   $p_{v,w} = 1, A_0 = \{4\}$ 



Output the set of all nodes activated  $A \leftarrow \bigcup_{t=0}^{T} A_t$ 

$$A_1 = A_2 =$$
$$A_3 = A_4 =$$

# Independent Cascade Model

#### Independent Cascade Model (Goldenberg, 2001)

 $\begin{array}{ll} \mbox{Initial set of active nodes } A_0 \\ \mbox{For } t = 1 \dots T \\ A_t \leftarrow \emptyset \\ \mbox{For } v \in A_{t-1} \\ \mbox{For } w \in neighbor \, (v) \mbox{ and } w \not\in \cup_{\tau=0}^t A_\tau \\ \mbox{If } rand(\cdot) < p_{v,w}, \mbox{ then} \\ A_t \leftarrow A_t \cup \{w\} \end{array}$ 

 $p_{v,w} = 1, A_0 = \{4\}$ 1 2 3 4 5 6 9 7 8 10 11

Output the set of all nodes activated  $A \leftarrow \bigcup_{t=0}^{T} A_t$ 

$$A_1 = \{2,3,5,6\}$$
 $A_2 = \{1,7,8,9\}$  $A_3 = \{10\}$  $A_4 = \{11\}$ 

# Poll I

- How many time steps are needed to achieve global cascade in the following example with p<sub>v,w</sub> = 1 and A<sub>0</sub> = {1}?
  A:2
  - B: 3
  - C:4
  - D: 5
  - E: None of the above
  - F: I don't know



### Independent Cascade Model Example with $p_{v,w} < 1$



### **Influence Propagation Models**

- Linear Threshold Model (M. Granovetter, 1978, T. Schelling, 1970, 1978)
  - Initial set of active nodes, Discrete time steps
  - Each node v has a threshold  $\theta_v$
  - Each edge has a weight  $b_{vw}$  indicating the influence of node v to node w

$$\sum_{v \in N(v)} b_{vw} \le 1$$

A node v becomes active when total weight of active neighbors exceeds threshold  $\theta_v$ 

$$\sum_{w \in N(v) \text{ and } w \text{ is active}} b_{vw} \ge \theta_v$$

### Linear Threshold Model Example



# Influence Maximization Problem

• How to select initial nodes  $A_0$  to maximize influence  $\sigma(A_0)$ , under the constraint that  $A_0$  has no more than K nodes

 $\max_{A_0} \sigma(A_0)$ <br/>s.t.  $|A_0| \le K$ 

The problem is NP-Hard (Kempe, Kleinberg & Tardos, 2003, 2005)

# Submodular Functions

- Submodular Functions
  - $f: 2^N \to \mathbb{R}$  is submodular if
    - For sets S, T where  $S \subset T, \forall v \notin T$

 $f(S \cup \{v\}) - f(S) \ge f(T \cup \{v\}) - f(T)$ 

• If  $f(S \cup \{v\}) \ge f(S), \forall S, \forall v$ , we say f is monotone

- Diminishing return (similar to concave function): Marginal value is decreasing as the set gets larger
  - Define marginal value of v given S as

$$f_S(v) = f(S \cup \{v\}) - f(S)$$

▶ *f* is submodular iff  $f_S(v) \ge f_T(v)$  for sets *S*, *T* where *S* ⊂ *T* 

### Submodular Functions

- Example: Sensor Coverage Problem
  - Similar to maximum coverage problem



# Greedy Algorithm for Problems with Submodularity

- Greedy algorithm leads to  $1 \frac{1}{e}$  approximation for submodular monotone function
  - For Maximum Coverage problem: Greedily pick the subset that covers most uncovered elements in each step
    - $U = \{1, 2, \dots, 6\}$
    - A = {A<sub>1</sub>, A<sub>2</sub>, ..., A<sub>N</sub>} is the set of subsets of U, i.e., A<sub>i</sub> ⊂ U
      A<sub>1</sub> = {1,3,5}, A<sub>2</sub> = {2,4,6}, A<sub>3</sub> = {1,6}, A<sub>4</sub> = {5,6}
      f: 2<sup>N</sup> → ℝ
      - □ f(S) where  $S \subset A$  is the number of elements in U that is covered by any  $A_i \in S$

# Greedy Algorithm for Problems with Submodularity

- Example: Sensor Coverage Problem
  - Similar to maximum coverage problem



- Theorem: For both LTM and ICM,  $\sigma(A_0)$  is a submodular function (Kempe, Kleinberg & Tardos, 2003)
- Also, it is easy to show that  $\sigma(A_0)$  is monotone
- So greedy algorithm is a  $1 \frac{1}{e}$  approximation for influence maximization problem

#### **Greedy Algorithm for Influence Maximization**

 $A_{0} \leftarrow \emptyset$ For *iter* = 1..*K* Select  $v = \underset{v' \in V \setminus A_{0}}{\operatorname{argmax}} (\sigma(A_{0} \cup \{v'\}) - \sigma(A_{0}))$  $A_{0} \leftarrow A_{0} \cup \{v\}$ 

- Under ICM, if you can only activate one node to trigger the propagation process, which node should be selected to maximize influence with  $p_{v,w} = 1$ ?
- If p<sub>v,w</sub> < 1 and you can choose two nodes, which nodes will be chosen following the greedy algorithm?



Under LTM, which 2 nodes will be chosen with the greedy algorithm?



 $\sigma(A_0)$  is the expected number of nodes being activated in the end

Which 2 nodes will be chosen?





#### Influence Maximization Problem

Discussion

Monte Carlo Tree Search

# Case Study: HIV Prevention Among Homeless Youth

#### Discussion

- What are the possible applications of the models and algorithms introduced today?
- How to extend the current problem definition of influence maximization problem to reflect some characteristics of real-world problems?

What are other significant problems that need to be solved based on the propagation model?



#### Influence Maximization Problem

Discussion

Monte Carlo Tree Search

# Case Study: HIV Prevention Among Homeless Youth

- General framework to make online decision in sequential decision making problems
  - E.g., online planning in MDPs, to determine game plays in Go, chess, video games etc
- Not only applicable to MDPs, but also other domains that cannot be modeled as MDPs
  - The idea of Q value can still be used

- MCTS for single player setting: online planning in an unknown environment
- You are now in some state, need to choose an action, but you know nothing about the environment
- Helper: a simulator tells you your available actions, and reward after you take the action



- Build a search tree node by node
  - Node: state; Edge: available actions
- Repeat: Select  $\rightarrow$  Expand  $\rightarrow$  Simulate  $\rightarrow$  Backpropagate



- Build a search tree node by node
  - Node: state; Edge: available actions
- Repeat: Select  $\rightarrow$  Expand  $\rightarrow$  Simulate  $\rightarrow$  Backpropagate



# Simplest MCTS

- In each iteration
  - Select: Choose the branch with the highest value
  - Expand:Add one node by randomly selecting an action
  - Simulate: Uniform random rollout
  - Backpropagate: update mean return (average accumulated reward) along the path
- Output: action correspond to branch with highest value at the root node after K iterations

### Monte Carlo Tree Search Example

Q:Assume the numbers in the nodes represent the mean return, which leaf node will be expanded when using the simplest MCTS?



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### Monte Carlo Tree Search Example

Q:Assume the following tree is built for the Atari game, the numbers in the nodes represent the total number of times we win / the total number of times we visit the state, which nodes will be updated in the backpropagation step?



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#### Monte Carlo Tree Search Example

Q:Assume the following tree is built for the Atari game, the numbers in the nodes represent the total number of times we win / the total number of times we visit the state, which nodes will be updated in the backpropagation step?



# Recap: Upper Confidence Bound in MAB

- UCBI Algorithm:
  - Always choose the arm with the highest upper confidence bound defined as  $\mu_{UB}^k = \widehat{\mu_k} + \sqrt{\frac{2 \ln t}{N(k)}}$
  - Intuition: If  $\mu_{UB}^k$  is large, either arm k is a good arm or N(k) is small (not enough data is gathered)
  - General principle: optimism in the face of uncertainty

## Monte Carlo Tree Search

- Upper Confidence Bounds for Trees (UCT)
  - For each node, keep track of estimated action value and visit count: Q(s, a) and N(s, a)
  - Select: Balance exploration vs exploitation:
    - If some actions never been chosen, randomly choose among them
    - Choose branch with highest Upper Confidence Bounds (UCB):

$$Q^{\oplus}(s,a) = Q(s,a) + c \sqrt{\frac{\ln N(s)}{N(s,a)}}$$

## Monte Carlo Tree Search

- Upper Confidence Bounds for Trees (UCT)
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  - Select: Balance exploration vs exploitation:
    - If some actions never been chosen, randomly choose among them
    - Choose branch with highest Upper Confidence Bounds (UCB):

$$Q^{\bigoplus}(s,a) = Q(s,a) + c \sqrt{\frac{\ln N(s)}{N(s,a)}}$$

back prop Game Ger! (

### Monte Carlo Tree Search Example

Q:Assume the numbers in the nodes represent the Q(s,a) / N(s,a), which leaf node will be expanded when using UCT with c = 10000?



## Monte Carlo Tree Search

#### More advanced MCTS

- Other advanced options:
  - Simulate: Terminate after  $T_0$  steps and estimate the reward
  - Expand:Add more nodes to the tree
  - Output: Optimal action at root node, as well as Q and N in the subtree corresponds to the optimal action
  - Initialize search tree with domain knowledge



#### Influence Maximization Problem

Discussion

- Monte Carlo Tree Search
- Case Study: HIV Prevention Among Homeless Youth









## **HIV Prevention Among Homeless Youth**



# **HIV Prevention Among Homeless Youth**

- Organize interventions among homeless youth.
  - > Try to raise awareness about HIV prevention practices.
  - Urge them to adopt safer behaviors.
  - Encourage them to spread the message among their social circles.



S.Ρ.Υ

safe place for youth



### **Resource Constraints with Homeless Shelters**

- Homeless shelters operate under resource constraints
  - Cannot intervene on every homeless youth themselves.
  - Rely on word-of-mouth effects among homeless youth.
  - Maximize number of youth who get informed about HIV.



# Influence Maximization Problem

- Given social network G and influence model I, choose K nodes to maximize expected influence spread
- Existing algorithms
  - With theoretical guarantees
    - Greedy [Kempe et al. 2003]
    - CELF [Leskovec et al. 2007]
    - > TIM [Borgs et al. 2012, Tang et al. 2014]
  - Efficient Heuristics
    - IRIE [Jung et al. 2011]
    - Sketch based heuristic [Cohen et al. 2015]



**Crucial Practical Challenge: Uncertainty** 

- Uncertainty in problem input
  - Uncertainty in social network structure



## **Crucial Practical Challenge: Uncertainty**

- Uncertainty in problem input
  - Uncertainty in social network structure
  - Uncertainty in specification of influence model



## **Crucial Practical Challenge: Uncertainty**

- Uncertainty in problem input
  - Uncertainty in social network structure
  - Uncertainty in specification of influence model
- Uncertainty during problem execution
  - Uncertainty about state of influence of nodes



Especially important if need to choose seed nodes sequentially





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## **Problem Definition**

- Dynamic Influence Maximization
  Under Uncertainty (DIME)
  - Input: A social network G = (V, E)
    - Consider uncertainty in social network structure
    - Each edge is associated with an existence probability u(e), or even a distribution
  - Influence propagation model
    - Similar to ICM but with nodes get multiple chances to influence their un-influenced neighbors
    - Each edge is associated with propagation probability p(e)



# **Problem Definition**

- Task
  - Choose a sequential adaptive plan (policy)
    - Picking a subset of nodes for M rounds
      - $\hfill\square\hfill\hf$
    - ▶ Size of each subset is *K* 
      - □ Maximum capacity of shelter
    - Maximize expected influence spread
      - $\hfill \hfill \hfill$
- Uncertainty about edge and state
  - Does not observe exact influence state
  - Can observe existence of edges adjacent to seed nodes

<b>→ B</b>
<b>E</b> ,,
F

#### **Theoretical Results**

# THEOREM I (AAMAS'I6A)

- DIME Problem is NP-Hard
- i.e., very hard to solve exactly

# THEOREM 2 (AAMAS'I6A)

- For any  $\epsilon$  > 0, it is impossible for an algorithm to guarantee a  $n^{-1+\epsilon}$  approximation to the full information optimal solution.
- i.e., very hard to approximate in polynomial time
- THEOREM 3 (AAMAS'I6A)
  - > The influence function of DIME is not adaptive sub-modular.
  - Thus, greedy alg. may not perform well empirically

#### Discussion

- Can we use greedy algorithm for influence maximization to solve DIME? What are the limitations of doing so?
- Formulate DIME as a POMDP. What are the states, actions, observations?
- Can we use Monte Carlo Tree Search to solve DIME? How? What are the potential issues?

## HEALER Algorithm Overview



# Key Idea I: Graph Partitioning

Choose a small subset of nodes in each cluster of nodes



## Key Idea 2: Graph Sampling

- We don't know the actual graph structure because of the uncertainty of edge existence
- Then draw a few samples!
- Choose subset of nodes that can be good in expectation for the sampled graphs

Key Idea 3: Monte-Carlo Tree Search + Multi-Armed bandit

- In each step, choose one node
- Maintain an MAB at each search tree node



#### **Evaluation**













## Poll 2

- Why greedy algorithm for influence maximization (referred to as the IM problem) cannot be directly applied to the problem of spreading HIV-related information among homeless youth (referred to as IS problem)?
  - A: Greedy algorithm has a good approximation bound but does not perform well empirically
  - B: In this IS problem, we need to select seed nodes in several batches sequentially, which is different from the IM problem
  - C:There are uncertainties in the IS problem
  - D:The greedy algorithm cannot scale
  - E: None of the above
  - F: I don't know

## Acknowledgment

The slides are prepared based on lecture slides of Leonid Zhukov and guest lecture of Amulya Yadav

#### References

- Using Social Networks to Aid Homeless Shelters: Dynamic Influence Maximization under Uncertainty (Links to an external site.)
- Influence Maximization in the Field:The Arduous Journey From Emerging to Deployed Application (Links to an external site.)
- Uncharted but not Uninfluenced: Influence Maximization with an Uncertain Network

#### References

- Maximizing the spread of influence through a social network (Links to an external site.)
- Submodular Functions: Extensions, Distributions, and Algorithms. A Survey (Links to an external site.)
- Information and Influence Propagation in Social Networks

# **Backup Slides**

**Propagation Process** 

Discuss: when would you adopt a recommendation from your friends?

How to model a social network and influence in the network?

# Submodular Functions

- Submodular Functions
  - ▶ Alternative definition: f is submodular iff  $\forall S, T$

 $f(S \cup T) + f(S \cap T) \le f(S) + f(T)$ 

 After-class exercise: Prove that these two definitions are equivalent

#### Submodular Functions

- Example: Team of defensive resources (ground vehicle and unman aerial vehicle)
  - $\land A = \{GV1, GV2, UAV1, UAV2, UAV3\}$
  - f(S) where  $S \subset A$  is described by

f(S) = g(#GV) + g(#UAV)



## Submodular Functions

- Example: Maximum Coverage problem
  - $U = \{1, 2, \dots, m\}$  is the set of elements
    - E.g., m = 6
  - ▶  $A = \{A_1, A_2, ..., A_N\}$  is the set of subsets of U, i.e.,  $A_i \subset U$ 
    - For example,  $A_1 = \{1,3,5\}, A_2 = \{2,4,6\}, A_3 = \{1,6\}, A_4 = \{5,6\}$
  - $\flat \ f: 2^N \to \mathbb{R}$ 
    - f(S) where S ⊂ A is the number of elements in U that is covered by any A<sub>i</sub> ∈ S

Greedy Algorithm for Problems with Submodularity

- Example: Team of defensive resources (ground vehicle and unman aerial vehicle)
  - $\land A = \{GV1, GV2, UAV1, UAV2, UAV3\}$
  - f(S) where  $S \subset A$  is described by

f(S) = g(#GV) + g(#UAV)



## Poll

- Let  $\theta_0$  = common threshold,  $N_0$  = common number of neighbors.  $b_{vw} = \frac{1}{N_0}$ . Consider the following three scenarios
  - $\bullet SI: \theta_0 = a, N_0 = c$
  - S2:  $\theta_0 = a + 0.1$ ,  $N_0 = c$
  - S3:  $\theta_0 = a$ ,  $N_0 = c + 1$
- When c > 1, what is ordering of the probability of getting global cascade following the LTM model under these three scenarios?
  - A:SI>=S2>=S3
  - ► B: S3>=S2>=S1
  - C: S2>=S1, S3>=S1, relationship between S2, S3 is unknown
  - D: None of the above
  - E: I don't know